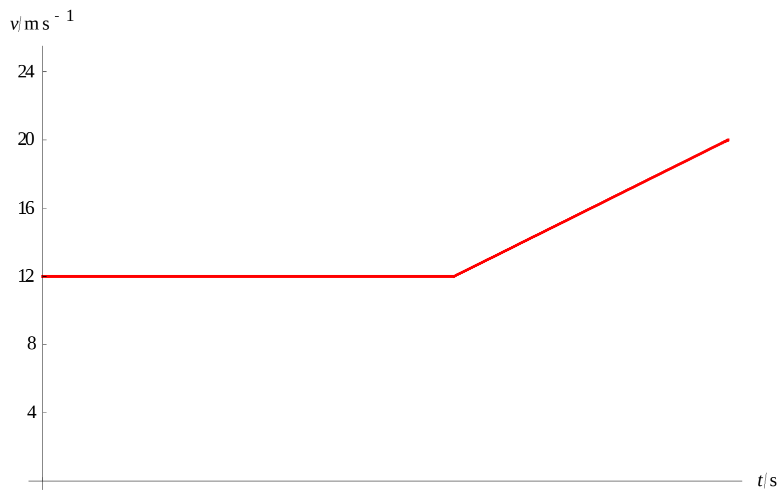
TOPIC A

Paper 2 Exam Questions

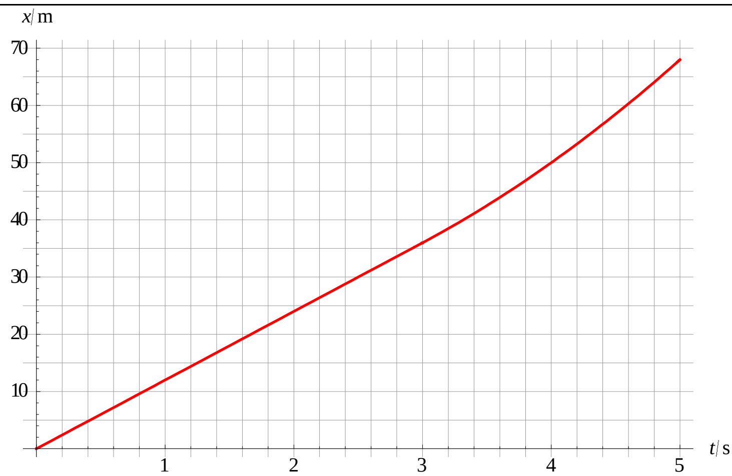
1.

A car moves along a straight line with constant speed 12 m s^{-1} for some time and then accelerates at 4.0 m s^{-2} to a speed 20 m s^{-1}. The total distance covered is 68 m. The graph shows how the speed of the car varies with time.



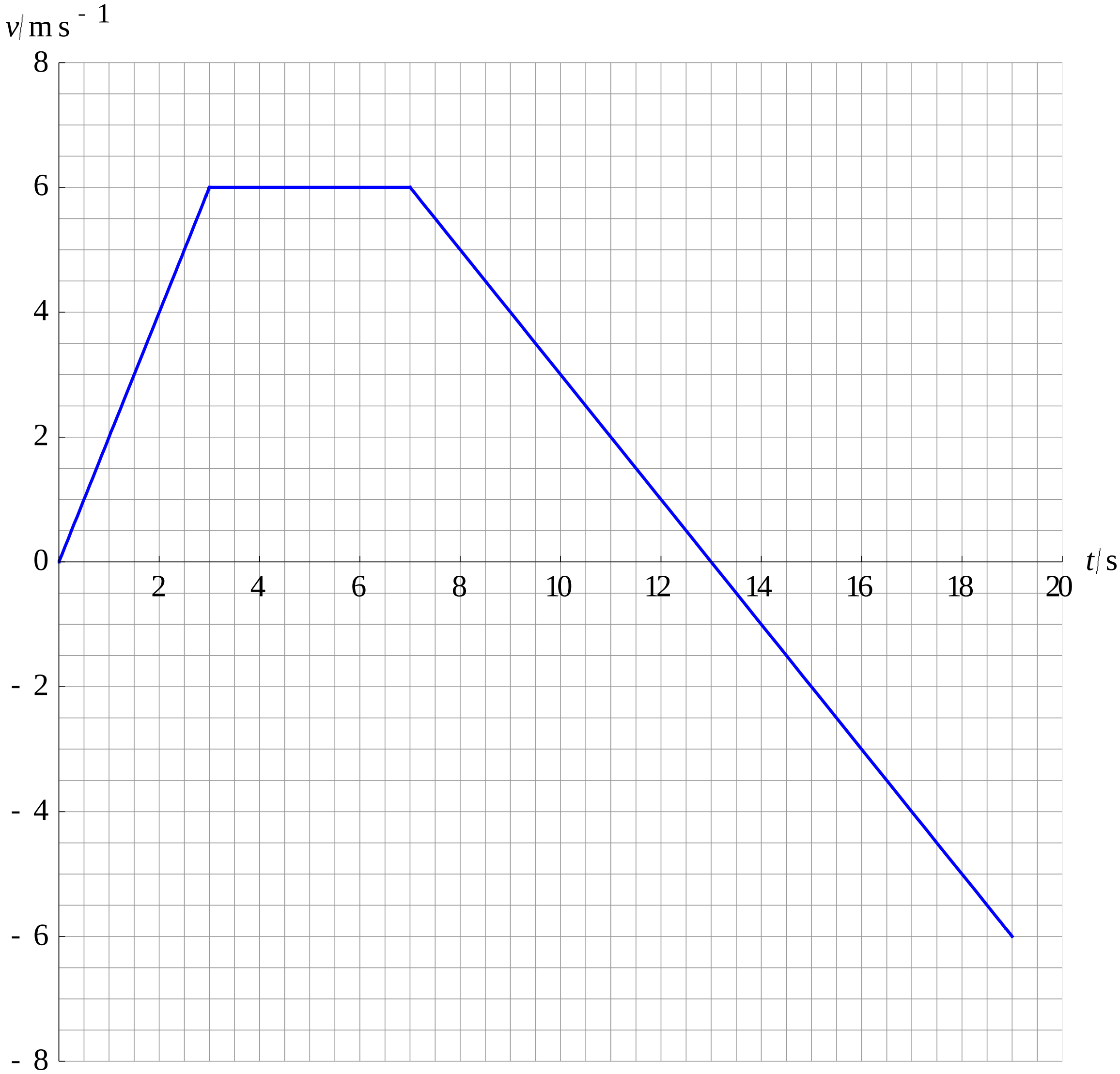
(a) Determine the total time of travel. **[3]**

(b) Sketch a graph to show the variation with time of the position of the car. Take the initial position to be zero. **[2]**

Question 1		Answers	Marks
a		$4.0=\dfrac{20-12}{T-\tau}\Rightarrow T-\tau=2.0\text{ s}$ ✓ $68=12\times(T-2)+\dfrac{12+20}{2}\times(T-\tau)=12\times(T-2)+\dfrac{12+20}{2}\times 2.0$ ✓ $T=5.0\text{ s}$ ✓	**3**
b		 Straight line up to 3 s ✓ Concave up curve for $3 < t < 5$ ✓	**2**

2.

The velocity of an object varies with time according to the graph.



(a) Draw the graph that shows how the position of this object varies with time. The initial position is zero. **[3]**

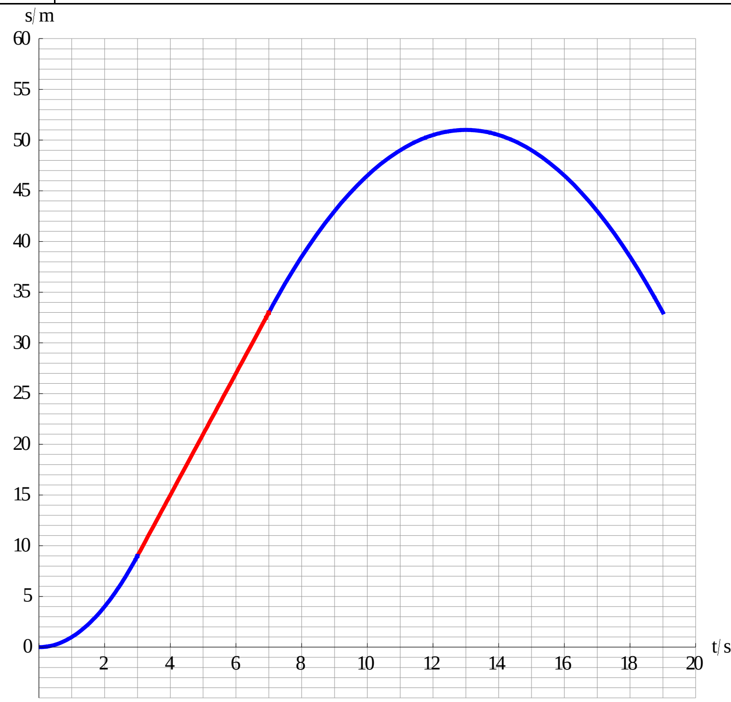
(b) Calculate, for this motion, the average

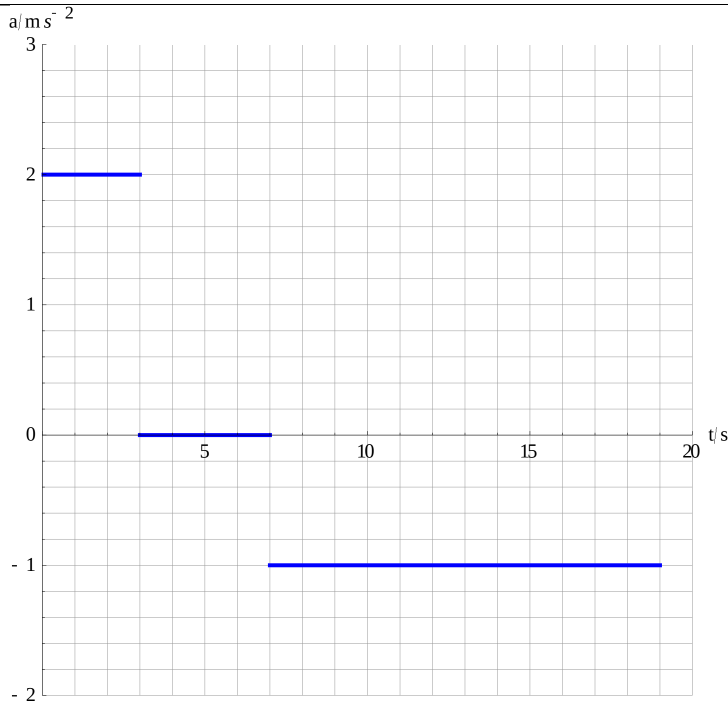
(i) velocity **[2]**

(ii) speed **[2]**

(iii) acceleration **[2]**

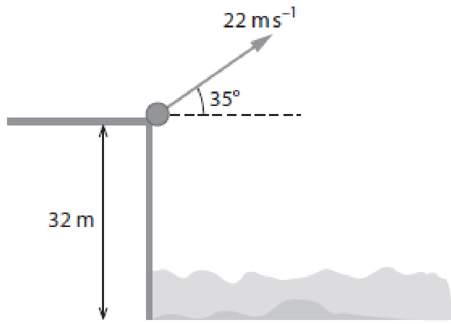
(c) Draw a graph that shows the variation with time of the acceleration of the body. **[2]**

Question 2		Answers	Marks
a		 ✓ for each feature (concave up initially, straight part, concave down)	3
b	i	$\frac{s_f - s_i}{T} = \frac{33-0}{19}$ ✓ $= 1.7\,\text{ms}^{-1}$ ✓	2
b	ii	$\frac{\text{distance travelled}}{T} = \frac{51+18}{19}$ ✓ $= 3.6\,\text{ms}^{-1}$ ✓	2
b	iii	$\frac{v_f - v_i}{T} = \frac{-6.0-0}{19}$ ✓ $= -0.32\,\text{ms}^{-2}$ ✓	2

c		 ✓✓	2

3.

A stone of mass 0.20 kg is thrown with speed 22 m s^{-1} from the edge of a cliff that is 32 m above the sea. The initial velocity of the stone makes an angle of 35° with the horizontal. Air resistance is neglected.



(a) (i) Determine the horizontal and vertical components of the initial velocity. **[2]**

(ii) Draw graphs showing the variation with time of the horizontal and vertical components of velocity until the stone hits the sea. (No numbers are required.) **[2]**

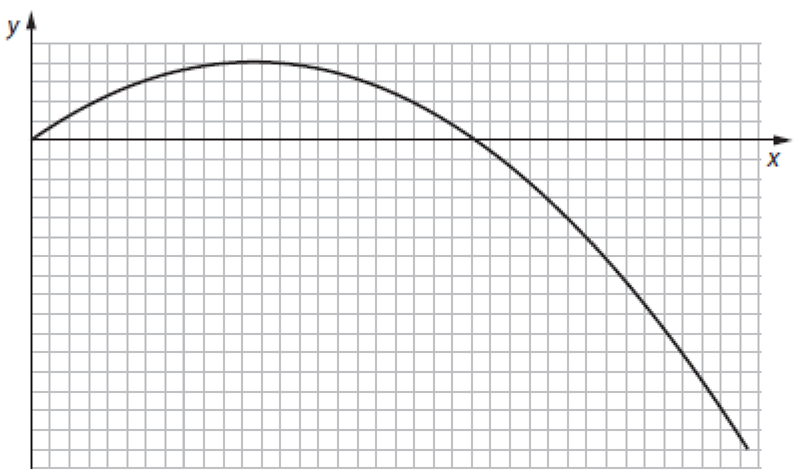
(b) (i) Calculate the maximum height above the cliff reached by the stone. **[3]**

(ii) State the net force on the stone at the highest point in its path. **[1]**

(c) (i) Using conservation of energy, determine the speed of the stone as it hits the sea. **[2]**

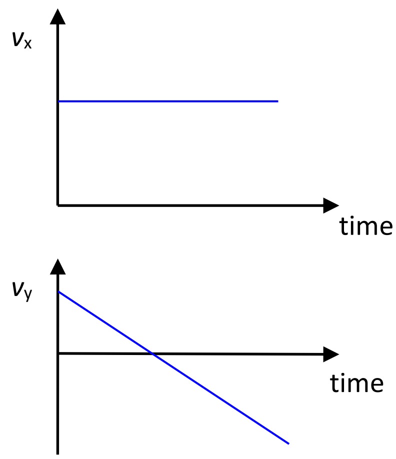
(ii) Hence or otherwise, determine the time it took the stone to reach the surface of the sea. **[2]**

The graph shows the path followed by this stone, until just before hitting the sea, in the absence of air resistance.



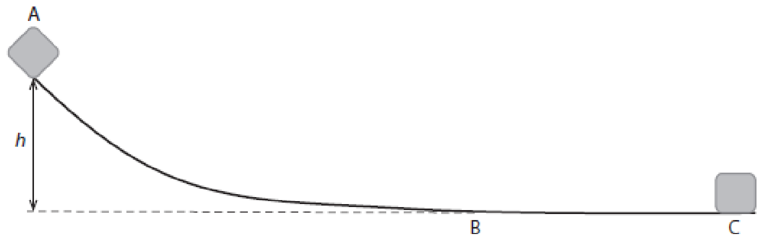
d (i) Draw the path of the stone in the presence of an air resistance force opposite to the velocity and proportional to the speed. **[3]**

(ii) State and explain one difference between your graph and the graph above. **[2]**

Question 3		Answers	Marks
a	i	$v_x = v\cos\theta = 22\times\cos 35^\circ = 18.0 \approx 18\,\text{ms}^{-1}$ ✓ $v_y = v\sin\theta = 22\times\sin 35^\circ = 12.6 \approx 13\,\text{ms}^{-1}$ ✓	2
a	ii	 ✓✓	2
b	i	At maximum height: $v_y{}^2 = 0 = u_y{}^2 - 2gy$ ✓ $y = \dfrac{u_y{}^2}{2g}$ ✓ $y = \dfrac{12.6^2}{2\times 9.8} = 8.1\,\text{m}$ ✓ ***OR*** $v_y = 0 = v\sin\theta - gt$ $12.6 - 9.8t = 0$ ✓ $t = 1.29\,\text{s}$ ✓ $y = 12.6\times 1.29 - \dfrac{1}{2}\times 9.8\times 1.29^2 = 8.1\,\text{m}$ ✓	3
b	ii	The force is the weight i.e. $F = 0.20\times 9.8 = 1.96 \approx 2.0\,\text{N}$ ✓	1
c	i	$\dfrac{1}{2}mu^2 + mgh = \dfrac{1}{2}mv^2$ hence $v = \sqrt{u^2 + 2gh}$ ✓ $v = \sqrt{u^2 + 2gh} = \sqrt{22^2 + 2\times 9.8\times 32} = 33.3 \approx 33\,\text{ms}^{-1}$ ✓	2
c	ii	$v^2 = v_x{}^2 + v_y{}^2 \Rightarrow v_y = -\sqrt{v^2 - v_x{}^2} = -\sqrt{33.3^2 - 18.0^2} = -28.0\,\text{m s}^{-1}$ ✓ $v_y = u_y\sin\theta - gt$ so $-28.0 = 12.6 - 9.8\times t$ hence $t = 4.1\,\text{s}$ ✓ ***OR*** Considering the motion from the very top: $y = 0 + \dfrac{1}{2}gt^2$ i.e. $8.1 + 32 = 0 + \dfrac{1}{2}gt^2$ $t = \sqrt{\dfrac{2\times 40.1}{9.8}} = 2.86\,\text{s}$ so total time is $t = 1.29 + 2.86 = 4.1\,\text{s}$ ✓	2
d	i	Lower max height ✓ shorter range ✓ sharper descent ✓	3

d	**ii**	Total energy will be reduced due to production of thermal energy✓ So at maximum height total energy is *mgh* plus the kinetic energy due to the horizontal component of velocity which has been reduced✓ will be less so maximum height will be less ***OR*** Range is horizontal velocity times time✓ horizontal velocity approaches zero✓ So range will be less ***OR*** The horizontal velocity component gets smaller and smaller✓ while vertical component approaches a constant value✓ Hence angle steeper	**2**

4.
In a factory blocks of ice slide down a smooth curved path AB and then on to a rough horizontal path starting at B.



The length of the curved path AB is s; the block of ice takes time t to move from A to B. The speed at B is v.

(a) Explain why, for the motion of the block from A to B:

(i) the formula $s = \frac{1}{2}gt^2$ does not apply. **[1]**

(ii) the formula $v = \sqrt{2gh}$ does apply. **[1]**

(b) A block of ice slides from A to B. The speed of the block at B is v_B = 4.8 m s^{-1}. Calculate the height h. **[1]**

(c) (i) The coefficient of dynamic friction between the block of ice and the rough surface BC is 0.45. The mass of the block is 25 kg. Show that the distance BC at which the block of ice is brought to rest is 2.6 m. **[3]**

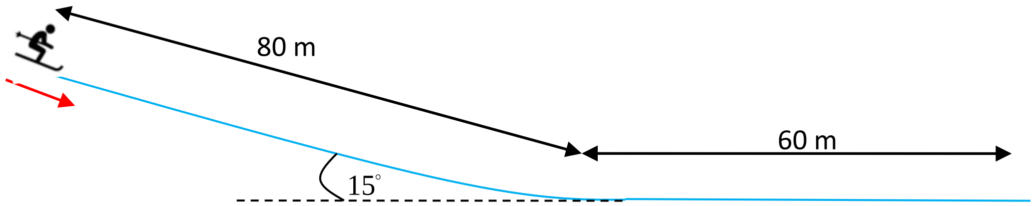
(ii) Calculate the time it takes the block of ice to cover the distance BC. **[2]**

(d) The factory also produces blocks of ice of mass 50 kg that slide down the same path starting at A. Predict, for this heavier block of ice, the speed at B and the stopping distance BC. (The coefficient of friction stays the same.) **[3]**

Question 4		Answers	Marks
a	**i**	The equation applies to straight line motion with acceleration *g*. Neither condition is satisfied here✓	**1**
a	**ii**	This equation is the result of energy conservation, so it does apply since there are no frictional forces present✓	**1**
b		From $v=\sqrt{2gh}$ we find $h=\frac{v^2}{2g}=\frac{4.8^2}{2\times 9.8}=1.176\approx 1.2\,\text{m}$ ✓	**1**
c	**i**	The kinetic energy at B is $E=\frac{1}{2}mv^2=\frac{1}{2}\times 25\times 4.8^2=288\,\text{J}$ ✓ The frictional force is $f=\mu_K N=\mu_K mg=0.45\times 25\times 9.8=110.25\,\text{N}$ ✓ The work done by this force is the change in the kinetic energy of the block and so $110.25\times d=288\Rightarrow d=2.61\approx 2.6\,\text{m}$ ✓	**3**
c	**ii**	The deceleration is $\frac{f}{m}=\frac{110.25}{25}=4.41\,\text{ms}^{-2}$ ✓ And so $0=4.8-4.41\times t$ giving 1.1 s for the time✓	**2**
d		The speed at B is independent of the mass✓ $fd=\frac{1}{2}mv^2\Rightarrow \mu_K mgd=\frac{1}{2}mv^2\Rightarrow d=\frac{v^2}{2\mu_K g}$ ✓ and so the distance is also independent of the mass✓	**3**

5.

A skier starting from rest, skis a distance of 80 m down a slope inclined at 15° to the horizontal. After reaching level ground she stops after travelling 60 m.



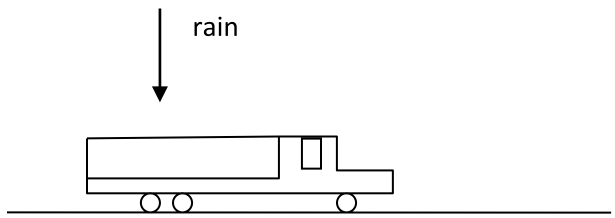
(a) Determine the coefficient of friction between the snow and the skis. **[5]**

(b) Calculate the speed of the skier when she reaches level ground. **[2]**

Question 5		Answers	Marks
a		The change in kinetic energy from start to finish is zero. Hence the net work done is zero✓ Along the slope the net work done is $mgd\sin 15^\circ - fd = mgd\sin 15^\circ - \mu mg\cos 15^\circ d$ ✓ since the frictional force is $f = \mu N = \mu mg\cos 15^\circ$ ✓ Along the flat track the net work is $-\mu mgs$ ✓ and so the net work is $mgd\sin 15^\circ - \mu mgd\cos 15^\circ - \mu mgs = 0$ giving $\mu = \dfrac{d\sin 15^\circ}{d\cos 15^\circ + s} = \dfrac{80\times\sin 15^\circ}{80\times\cos 15^\circ + 60} = 0.15.$ ✓	5
b		For the flat track, $0 - \dfrac{1}{2}mv^2 = -\mu mgs$ and so we find $\dfrac{1}{2}v^2 = \mu gs$ ✓ and so $v = \sqrt{2\times 0.15\times 9.8\times 60} = 13.3 \approx 13\,\mathrm{ms}^{-1}$ ✓	2

6.

(a) A toy truck of mass 2.0 kg moves with constant speed 1.8 m s^{-1} on a horizontal road. When it starts to rain the truck begins to fill with water at a rate of $0.10\,\text{kg}\,\text{s}^{-1}$.



The truck begins to fill with water at t = 0 and completely fills after time 10 s.

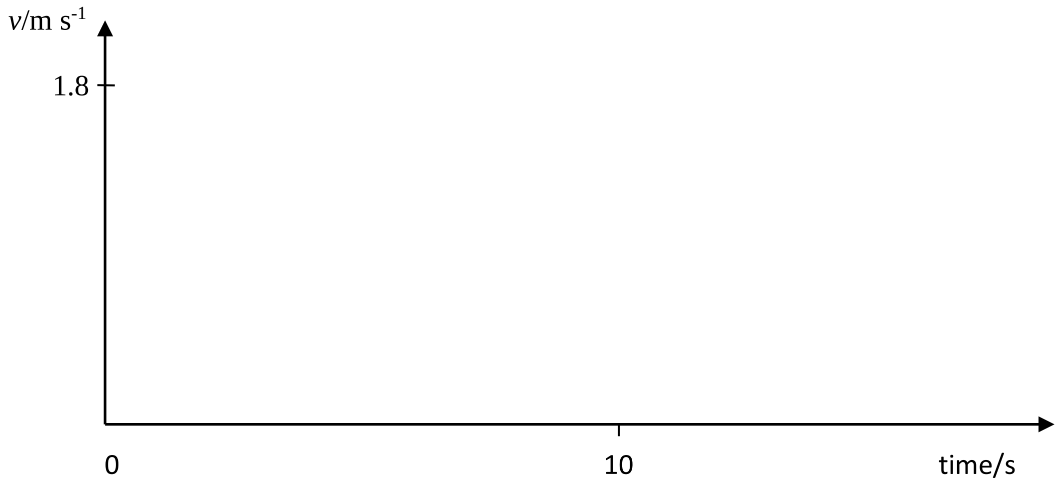
(i) Suggest why the momentum of the truck-water system remains constant. **[2]**

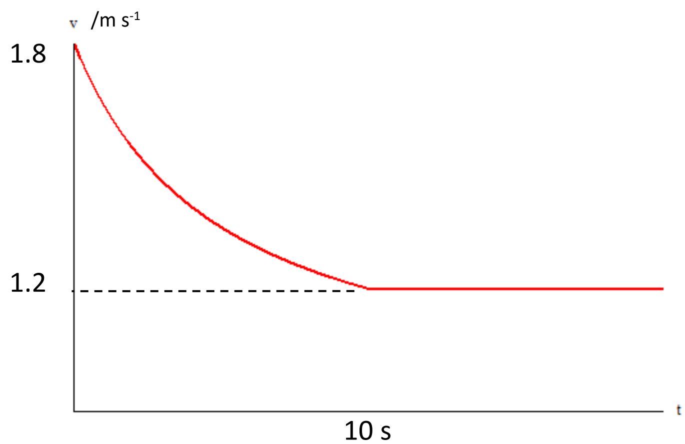
(ii) State an expression for the total mass of the truck after time t (< 10 s) **[1]**

(iii) Find an expression for the speed v of the truck after time t (< 10 s). **[2]**

(b) On the axes draw a graph to show the variation with time of the speed v of the truck.

[2]



Question 6		Answers	Marks
a	**i**	There is no external force acting on the truck✓ In the horizontal direction✓	**2**
a	**ii**	$2.0+0.10t$ ✓	**1**
a	**iii**	$2.0\times1.8=(2.0+0.10\times t)v$ ✓ $v=\dfrac{3.6}{2.0+0.10\times t}$ ✓	**2**
b		 Curve for $t < 10$ s✓ Constant for $t > 10$ s ✓	**2**

7.

(a) State the law of conservation of mechanical energy. **[1]**

A battery toy car of mass 0.250 kg is made to move up an inclined plane that makes an angle of 30° with the horizontal. The car starts from rest and its motor provides a constant acceleration of 4.0 m s^{-2} for 5.0 s. The motor is then turned off. (Use g = 10 m s^{-2}.)

(b) (i) Find the distance travelled in the first 5 s. **[1]**

(ii) Find the furthest the car gets on the inclined plane. **[3]**

(iii) Calculate when the car returns to its starting position. **[3]**

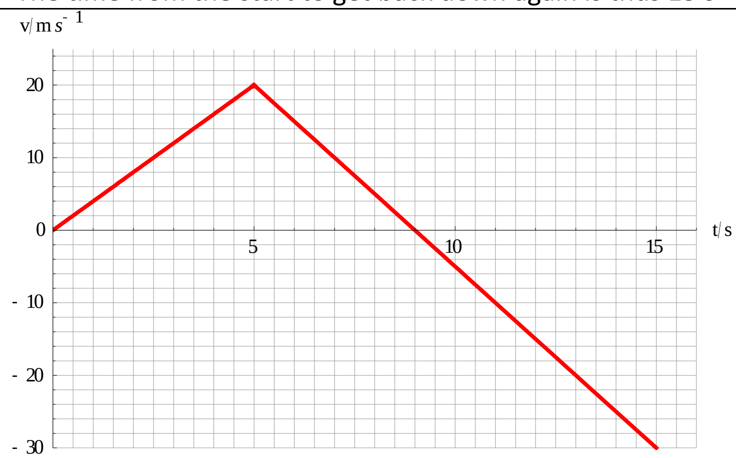
(c) Sketch a graph of the velocity as a function of time. **[2]**

(d) (i) On the same axes, sketch a graph of the kinetic energy and potential energy of the car as a function of the distance travelled. **[5]**

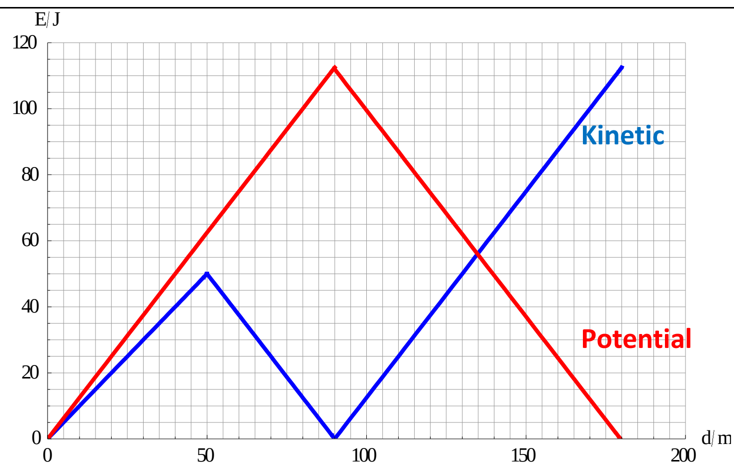
(ii) State and explain the periods in the car's motion in which its mechanical energy is conserved. **[2]**

(e) Estimate the average power developed by the car's motor. **[3]**

<table>
<tr><th colspan="2">Question 7</th><th>Answers</th><th>Marks</th></tr>
<tr><td>a</td><td></td><td>In the absence of resistive forces, the total mechanical energy of a system stays constant✓</td><td>1</td></tr>
<tr><td>b</td><td>i</td><td>From $s=\frac{1}{2}at^2$ we find $s=\frac{1}{2}\times 4.0\times 5.0^2=50\,\mathrm{m}$ ✓</td><td>1</td></tr>
<tr><td>b</td><td>ii</td><td>At 5.0 s the speed acquired is $v=at=4.0\times 5.0=20\,\mathrm{m\,s^{-1}}$ ✓
From then on the acceleration becomes a deceleration of
$a=g\sin\theta=10\times 0.5=5.0\,\mathrm{m\,s^{-2}}$ ✓
Then from $v^2=u^2-2as$ we find $0=20^2-2\times 5.0\times s$ giving $s=\frac{20^2}{2\times 5.0}=40\,\mathrm{m}$.
The total distance up the plane is thus 90 m✓</td><td>3</td></tr>
<tr><td>b</td><td>iii</td><td>$90=\frac{1}{2}\times 5.0\times t^2$ giving $t^2=\frac{180}{5.0}=36\Rightarrow t=6.0\,\mathrm{s}$ ✓
The car took 5.0 s to get to the 50 m up the hill. The remaining 40 m were covered in $0=20-5.0\times t\Rightarrow t=4.0\,\mathrm{s}$ ✓
The time from the start to get back down again is thus 15 s✓</td><td>3</td></tr>
<tr><td>c</td><td></td><td>



</td><td>2</td></tr>
<tr><td>d</td><td>i</td><td>

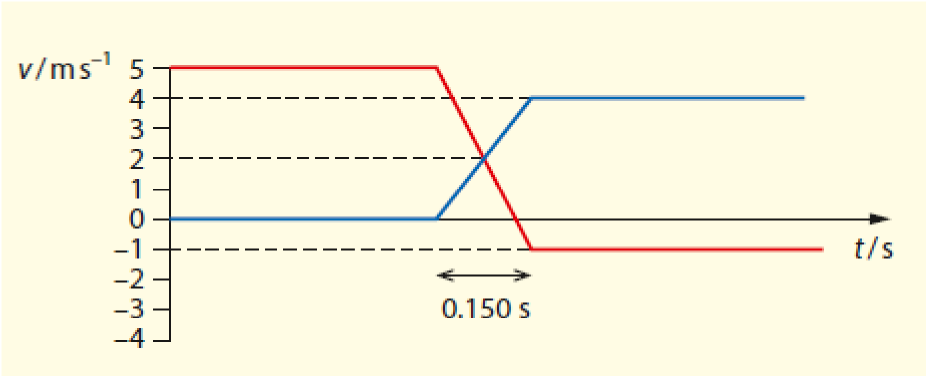


$E_P=mgh=mgd\sin\theta=0.250\times 10\times d\times\frac{1}{2}=1.25d$ for $0\le d\le 90$ ✓</td><td>5</td></tr>
</table>

		$E_P = 112.5 - 1.25d$ for $90 \le d \le 180$ ✓ $E_K = \frac{1}{2}mv^2 = \frac{1}{2}m2ad = \frac{1}{2} \times 0.250 \times 2 \times 4 \times d = d$ for $0 \le d \le 50$ ✓ $E_K = 50 - \frac{1}{2}m2(\frac{g}{2})d = 50 - 1.25d$ for $50 \le d \le 90$ ✓ $E_K = \frac{1}{2}m2(\frac{g}{2})d = 1.25d$ for $90 \le d \le 180$ ✓	
d	**ii**	The mechanical energy (kinetic plus potential) will be conserved when there are no external forces acting on the car (other than gravity) ✓ i.e. after the first 5.0 s or after the 50 m ✓	**2**
e		Kinetic energy at 5 s: $\frac{1}{2}mv^2 = \frac{1}{2} \times 0.250 \times 20^2 = 50$ J ✓ Potential energy at 5 s: $mgh = 0.250 \times 10 \times 25 = 62.5$ J ✓ Average power: $\frac{50 + 62.5}{5.0} = 22.5$ W ✓ ***OR*** Force exerted by motor: $F - mg\sin\theta = ma \Rightarrow F = ma + mg\sin\theta = 0.250 \times (4.0 + 10 \times \frac{1}{2}) = 2.25$ N ✓ Average power: $\bar{P} = F\frac{u+v}{2}$ ✓ $\bar{P} = 2.25 \times \frac{0+20}{2} = 22.5$ W ✓	**3**

8.

A train carriage of mass 800 kg moving at 5.0 m s^{-1} collides with a stationary carriage of mass 1200 kg. Both carriages are equipped with buffers. The graph shows how the velocities of the carriages vary before, during and after the collision.



Use the graph to

(a) show that the collision is elastic, **[2]**

(b) (i) calculate the impulse delivered to the heavier carriage, **[2]**

(ii) calculate the average force the carriages exert on each other, **[2]**

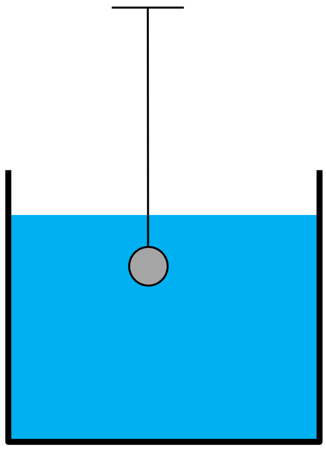
(c) predict how the answers to (b) would change if the buffers were stiffer but velocities stayed unchanged, **[2]**

(d) (i) calculate the total kinetic energy of the carriages when both have the same speed, **[1]**

(ii) compare the answer to (i) to the total final kinetic energy and discuss. **[2]**

Question 8		Answers	Marks
a		Initial KE: $\frac{1}{2}\times 800\times 5.0^2 = 1.0\times 10^4$ J ✓ Final KE: $\frac{1}{2}\times 800\times 1.0^2 + \frac{1}{2}\times 1200\times 4.0^2 = 1.0\times 10^4$ J ✓	**2**
b	**i**	$J = \Delta p$ ✓ $J = 1200\times 4.0 - 0 = 4.8\times 10^3$ N s ✓	**1**
b	**ii**	$F = \frac{\Delta p}{\Delta t}$ $F = \frac{4.8\times 10^3}{0.150} = 3.2\times 10^4$ N on the 1200 kg carriage✓ And so, the same on the other in the opposite direction✓	**2**
c		Impulse is the same✓ Force is larger since time interval will be less✓	**2**
d	**i**	$\frac{1}{2}\times 800\times 2.0^2 + \frac{1}{2}\times 1200\times 2.0^2 = 4.0\times 10^3$ J ✓	**1**
d	**ii**	It is less✓ The rest of the energy is elastic energy in the buffers✓	**2**

9.
A small ball of radius 2.0 cm is attached to a vertical string and is fully submerged in a fluid.



The density of the ball is 2800 kg m^{-3} and that of the fluid is 820 kg m^{-3}.

(a) Calculate the tension in the string. **[3]**

(b) The string is cut and the ball moves in the liquid eventually achieving terminal speed.

(i) Calculate the initial acceleration of the ball. **[2]**
(ii) Explain why the ball will reach terminal speed. **[2]**

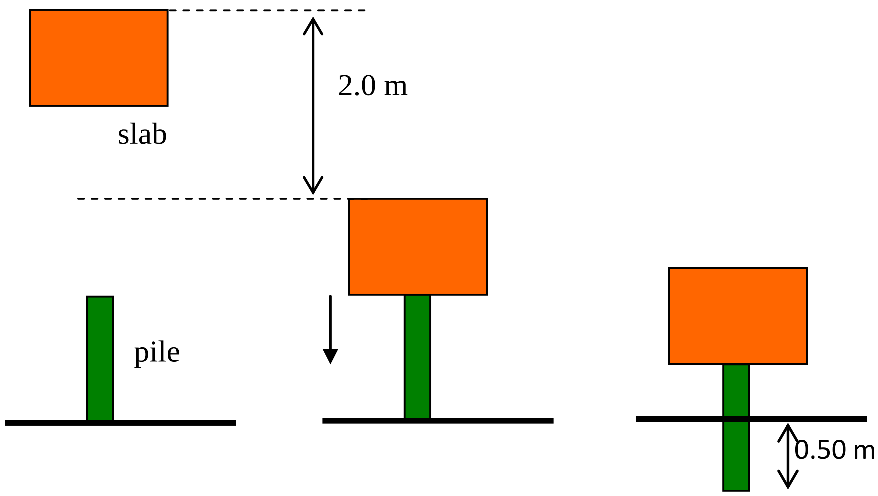
The viscosity of the liquid is 0.54 Pa s.

(c) Determine the terminal speed. **[2]**

(d) Describe the energy transfers taking place when the ball moves at terminal speed. **[1]**

Question 9		Answers	Marks
a		$T+B=mg \Rightarrow T=mg-B=\rho_{\text{ball}}Vg-\rho_{\text{liquid}}Vg$ ✓ $T=(\rho_{\text{ball}}-\rho_{\text{liquid}})\dfrac{4\pi}{3}r^3g$ ✓ $T=(2800-820)\times\dfrac{4\pi}{3}\times(2.0\times10^{-2})^3\times9.8=0.65\text{ N}$ ✓	3
b	i	Initial net force is 0.65 N ✓ $a=\dfrac{0.65}{2800\times\dfrac{4\pi}{3}\times(2.0\times10^{-2})^3}=6.9\text{ m s}^{-2}$ ✓	2
b	ii	As soon as the ball begins to move it will experience a drag force opposite to the velocity✓ As the speed increases the drag force increases as well eventually becoming equal to $mg-B$ /equal to 0.65 N✓	2
c		$6\pi\eta rv=0.65\text{ N}\Rightarrow v=\dfrac{0.65}{6\pi\times0.54\times(2.0\times10^{-2})}$ ✓ $v=3.2\text{ m s}^{-1}$ ✓	2
d		Gravitational potential energy is getting transferred to thermal energy in the ball and the liquid✓	1

10.
A slab of mass 54 kg falls from rest from a height of 2.0 m above a pile of mass 18 kg as shown in the diagram.



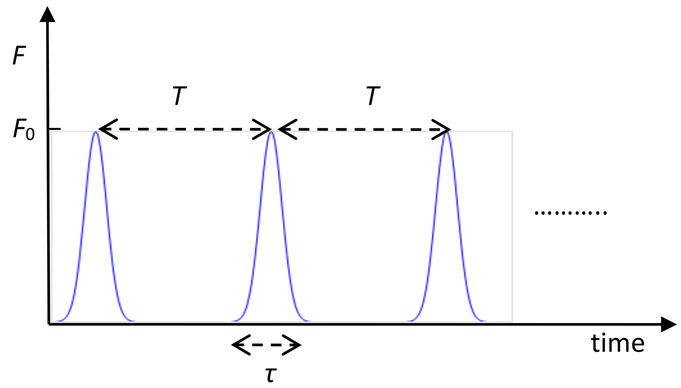
(a) Calculate the speed of the slab as it impacts the pile. **[1]**
(b) Calculate the common speed at which the pile and the slab move immediately after impact. **[2]**
(c) The pile is driven to a depth of 0.50 m in the ground. Calculate the average upward force the ground exerted on the pile while the pile is driven into the ground. **[3]**

Question 10		Answers	Marks
a		$v=\sqrt{2\times 9.8\times 2.0}=6.26\approx 6.3\,\text{ms}^{-1}$ ✓	1
b		$54\times 6.26=(54+18)\times u$ ✓ $u=\dfrac{54\times 6.26}{54+18}=4.70\approx 4.7\ \text{ms}^{-1}$ ✓	2
c		The work done by the net force equals the change in kinetic energy and so $0-\dfrac{1}{2}(54+18)\times 4.70^2=F\times 0.50\times(-1)$ ✓ hence $F=1590\approx 1600\,\text{N}$ ✓ The net force is the reaction from the ground minus the weight and so $1590=R-72\times 9.8\Rightarrow R=2296\approx 2.3\,\text{kN}$ ✓	3

11.

(a) Outline what is meant by the impulse of a force. **[2]**

The net force F applied to a body varies with time as shown in the graph.



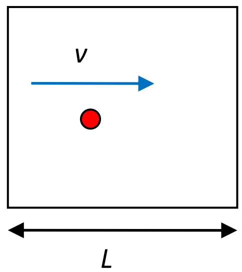
The area under each curve is J.

(b)

(i) Determine the average force exerted on the body during a time interval of τ. **[1]**

(ii) Show that the average force over an interval of time NT, where N is very large is $\frac{J}{T}$. **[2]**

(c) The graph might be used to model the force exerted on a wall of the container by a gas molecule. The diagram shows a molecule of mass m moving with horizontal velocity v. The molecule collides with the walls elastically.



(i) Calculate the impulse delivered to the right wall of the container by the molecule. **[2]**

(ii) Write down an expression for T, the time between consecutive impacts with the right wall. **[1]**

(iii) Determine, using the result in (b) (ii), the average force this molecule exerts on the right wall. **[2]**

Question 11		Answers	Marks
a		Impulse is the product of the force times the time for which the force acts✓ It equals the change in momentum✓	2
b	i	$\bar{F}=\frac{J}{\tau}$ ✓	1
b	ii	A time of *NT* includes *N* impulses and so the impulse supplied is *NJ*✓ The average force is then $\frac{NJ}{NT}=\frac{J}{T}$ ✓	2
c	i	Collisions are elastic so molecule bounces with same speed✓ $J=mv-(-mv)=2mv$ ✓	2
c	ii	$T=\frac{2L}{v}$ ✓	1
c	iii	$\frac{J}{T}$, i.e. $\frac{2mv}{T}=\frac{2mv}{\frac{2L}{v}}$ ✓ $F=\frac{mv^2}{L}$ ✓	2

12.
A toy helicopter has mass m = 0.30 kg and blade rotors of radius R = 0.25 m. As the blades turn, the air under the blades is pushed downwards with speed v. The density of air is ρ = 1.2 kg m^{-3}.

(a) Explain how the lift force on the helicopter is produced. **[2]**

(b) (i) Show that the force that the rotor blades exert on the air is $\rho\pi R^2v^2$. **[3]**
(ii) Hence estimate the speed v of air when the helicopter just hovers. **[2]**

(c) Determine the average power supplied to the air when the helicopter just hovers as in (b). **[2]**

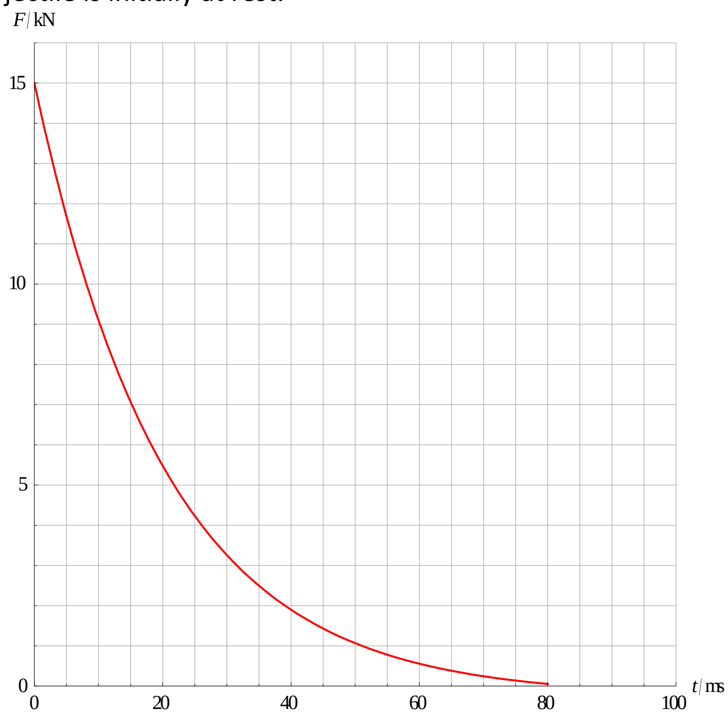
(d) The rotor blades now move faster pushing air downwards at a speed double that found in (a). The helicopter is raised vertically a distance of 12 m.

Estimate,
(i) the acceleration of the helicopter, **[3]**
(ii) the time needed to raise the helicopter 12 m. **[2]**

Question 12		Answers	Marks
a		The rotor exerts a downward force on the air✓ The air exerts and equal and opposite force on the rotor✓	**2**
b	**i**	In 1 second the mass of air that will move down is $\rho(\pi R^2 v)$ ✓ Δp in 1 s is $\rho(\pi R^2 v)v = \rho\pi R^2 v^2$ ✓ and from $F = \dfrac{\Delta p}{\Delta t}$ this is the force✓	**3**
b	**ii**	$\rho\pi R^2 v^2 = mg$ ✓ $v = \sqrt{\dfrac{mg}{\rho\pi R^2}} = \sqrt{\dfrac{0.30\times 9.8}{1.2\times\pi\times 0.25^2}} = 3.53 \approx 3.5\,\mathrm{ms^{-1}}$ ✓	**2**
c		In 1 second the rotor forces air of mass $\rho(\pi R^2 v)$ to acquire speed and so the kinetic energy supplied in 1 second i.e. the average power supplied to air, is $\dfrac{1}{2}\rho(\pi R^2 v)v^2 = \dfrac{1}{2}\rho\pi R^2 v^3$ ✓ So $\bar{P} = 5.18 \approx 5.2$ W ✓	**2**
c	**i**	The lift force is now 4 times larger since $F \propto v^2$ ✓ The net force is then 3*mg* since initially the lift force equaled the weight✓ $a = 3g$ ✓	**3**
c	**ii**	$s = \dfrac{1}{2}(3g)t^2 \Rightarrow t = \sqrt{\dfrac{2s}{3g}}$ ✓ $t = \sqrt{\dfrac{2\times 12}{3\times 9.8}} = 0.90$ s ✓	**2**

13.

A launcher exerts a force on a projectile of mass 2.0 kg that varies with time according to the graph. The projectile is initially at rest.



(a) Sketch a graph to show how the speed of the projectile varies with time. No numbers are required on the axes. **[2]**

The projectile leaves the launcher in 80 ms. The impulse delivered to the projectile is 285 N s.

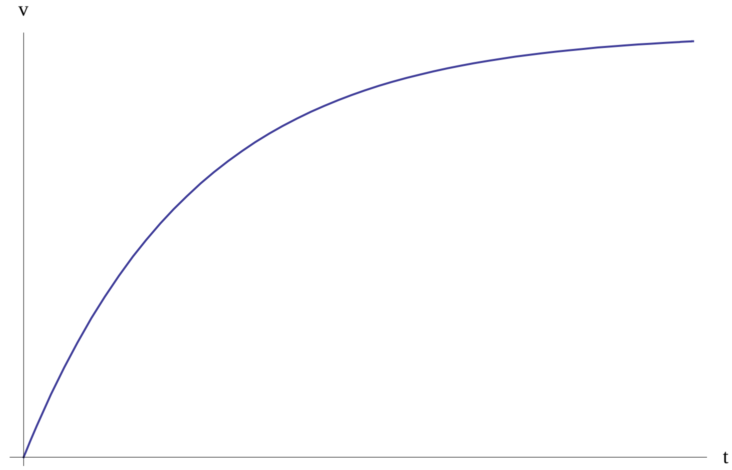
(b) (i) State the area under the curve from $t = 0$ to $t = 80$ ms. **[1]**

(ii) Estimate the average force exerted on the projectile. **[1]**

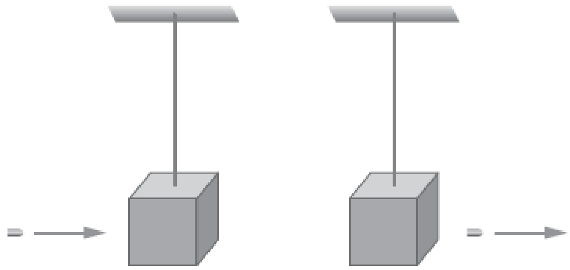
(iii) Determine the speed of the projectile as it exits the launcher. **[1]**

(c) Calculate the average power delivered to the projectile by the launcher. **[2]**

(d) The average speed of the projectile in the launcher is 91 m s^{-1}. Determine the length of the launcher. **[1]**

Question 13		Answers	Marks
a		 Correct curvature✓ Gradient approaching zero✓	2
b	i	285 N s ✓	1
b	ii	$\overline{F} = \frac{\Delta p}{\Delta t} = \frac{285}{80\times10^{-3}} = 3.56\times10^{3} \approx 3.6\times10^{3}$ N ✓	1
b	iii	$\Delta p = mv - 0 \Rightarrow v = \frac{285}{2} = 142.5 \approx 140$ m s^{-1} ✓	1
c		KE leaving launcher $\frac{1}{2}mv^2 = \frac{1}{2}\times2.0\times142.5^2 = 2.03\times10^4$ J ✓ Average power $\frac{2.03\times10^4}{80\times10^{-3}} = 2.5\times10^5$ W ✓ ***OR*** $\overline{P} = \overline{F}\frac{u+v}{2}$ ✓ $\overline{P} = 3.56\times10^3\times\frac{0+142.5}{2} = 2.5\times10^5$ W ✓	2
d		$91\times0.080 = 7.3$ m ✓	1

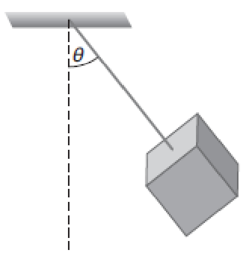
14.
A bullet of mass 0.090 kg is shot at a wooden block of mass 1.20 kg that is hanging vertically at the end of a string.



The bullet enters the block with speed 130 m s^{-1} and leaves it with speed 90 m s^{-1}. The mass of the block does not change appreciably because of the hole made by the bullet.

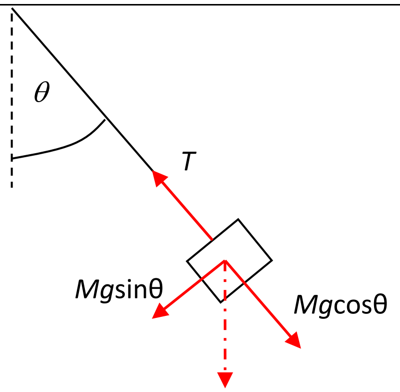
(a) (i) Calculate the magnitude of the impulse delivered to the bullet. **[2]**
(ii) Show that the initial velocity of the block is 3.0 m s^{-1}. **[1]**
(iii) Estimate the loss of kinetic energy in the bullet-block system. **[2]**

As a result of the impact, the block is displaced. The maximum angle that the string makes with the vertical is θ. The length of the string is 0. 80 m.

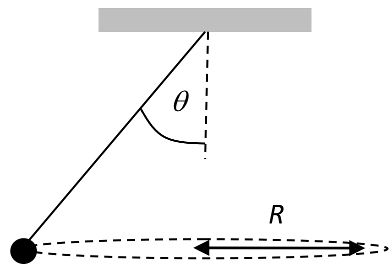


(b) (i) Show that $\theta \approx 65^{\circ}$. **[3]**
(ii) Calculate the net force on the block when $\theta \approx 65^{\circ}$. **[2]**
(iii) Suggest whether the block is in equilibrium when $\theta \approx 65^{\circ}$. **[2]**

(c) The block now swings like a pendulum. Calculate the tension in the string when the string becomes vertical. **[3]**

Question 14		Answers	Marks
a	i	$\lvert\Delta p\rvert = \lvert mv - mu\rvert = \lvert 0.090\times 90 - 0.090\times 130\rvert$ ✓ $\lvert\Delta p\rvert = 3.6\text{ N s}$ ✓	2
a	ii	$mu = mv + Mw$ ✓ $0.090\times 130 = 0.090\times 90 + 1.2\times w$ ✓ ***OR*** $\Delta p_{\text{block}} = +3.6\text{ N s}$ ✓ $w = \frac{3.6}{1.2}\text{ m s}^{-1}$ ✓ $w = 3.00\text{ m s}^{-1}$	2
a	iii	$\frac{1}{2}\times 0.090\times 130^2 - \frac{1}{2}\times 0.090\times 90^2 - \frac{1}{2}\times 1.2\times 3.0^2$ ✓ $= 390.6 \approx 390\text{ J}$ ✓	2
b	i	$\frac{1}{2}Mw^2 = Mgh = Mg(L - L\cos\theta)$ ✓ $\frac{w^2}{2gL} = 1 - \cos\theta$ ✓ $\cos\theta = 1 - \frac{w^2}{2gL} = 1 - \frac{3.0^2}{2\times 9.8\times 0.80} = 0.4260 \Rightarrow \theta = 64.78^\circ \approx 65^\circ$ ✓	3
b	ii	 $T - Mg\cos\theta = \frac{Mv^2}{L} = 0$ since $v = 0$ ✓ Net force $Mg\sin\theta = 1.2\times 9.8\times \sin 64.78^\circ = 10.6 \approx 11\text{ N}$ ✓	2
b	iii	The net force is not zero ✓ So the block is not in equilibrium ✓	2
c		$T - Mg = M\frac{w^2}{L} \Rightarrow T = M\frac{w^2}{L} + Mg$ ✓ $w = 3.00\text{ m s}^{-1}$ ✓ $T = 1.2\times\frac{3.0^2}{0.80} + 1.2\times 9.8 = 25.3 \approx 25\text{ N}$ ✓	3

15.
A ball is attached to a string and moves on a horizontal circular path with constant speed.



(a) Students claim that the ball is in equilibrium because the speed is constant. Explain why the students are not correct. [2]

(b) Draw and label the forces on the ball. [2]

(c)

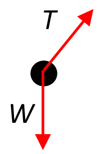
(i) Show that $\tan\theta = \frac{v^2}{gR}$ where v is the speed and R the radius of the circular path. [3]

(ii) Suggest why it impossible for the ball to rotate with the string horizontal. [1]

The following data are available:

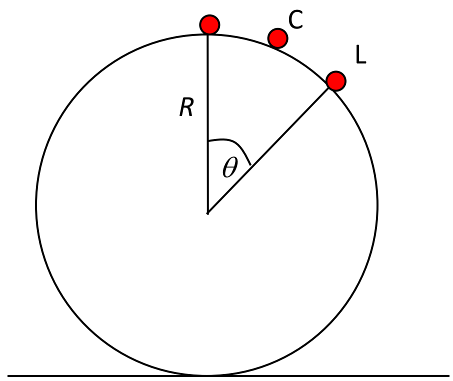
Mass of ball	2.0 kg
Radius of path	0.80 m
Speed v	6.2 m s^{-1}

(d) Calculate the tension in the string. [3]

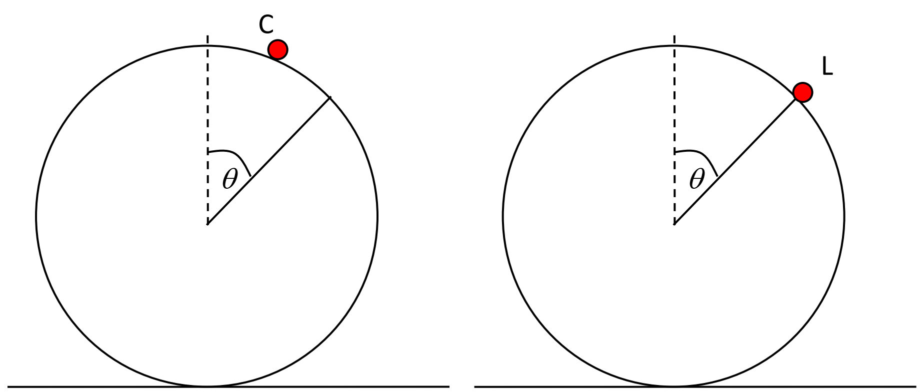
Question 15		Answers	Marks
a		The velocity is changing because the direction is ✓ Hence there is a net force, so no equilibrium ✓	**2**
b		 *W*eight✓ *T*ension, vertical component to match weight✓	**2**
c	**i**	$T\cos\theta = Mg$ ✓ $T\sin\theta = \frac{Mv^2}{R}$ ✓ Divide: $\frac{T\sin\theta}{T\cos\theta} = \frac{\frac{Mv^2}{R}}{Mg}$ ✓ To get result	**3**
c	**ii**	The speed would have to be infinite so impossible ✓ ***OR*** The weight could not be balanced so impossible ✓	**1**
d		$T\cos\theta = Mg \Rightarrow T = \frac{Mg}{\cos\theta}$ ✓ $\tan\theta = \frac{v^2}{gR} = \frac{6.2^2}{9.8\times 0.80} \Rightarrow \theta = 78.47^\circ$ ✓ $T = \frac{2.0\times 9.8}{\cos 78.47} = 98\text{ N}$ ✓	**3**

16.

A small ball is placed at the top of a sphere of radius R. The ball is given the slightest of pushes, so it begins to move. The ball will lose contact with the sphere at position L. Position C is an intermediate position.



(a) Draw, on the diagrams, the forces on the ball at positions C and L. [3]

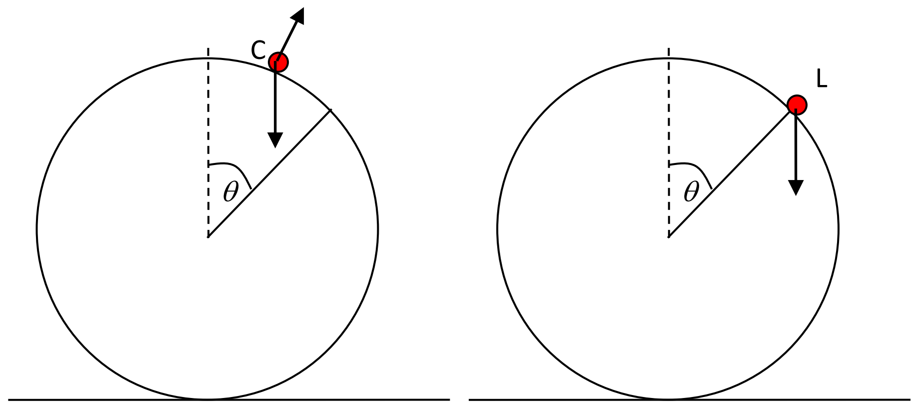


(b)

(i) Show that the speed of the ball at L is given by $v^2 = 2gR(1-\cos\theta)$. [2]

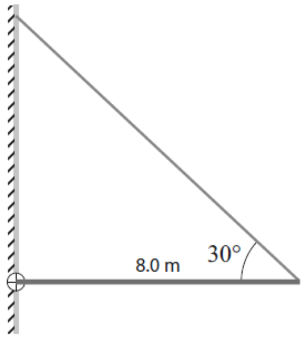
(ii) Determine the angle θ at position L where the ball leaves the sphere. [3]

(c) The sphere is replaced by another, larger sphere. Suggest the effect of this change, if any, on the answer to (b) (ii). [2]

Question 16		Answers	Marks
a		 Weight ✓ Normal at C ✓ No normal at L ✓	**3**
b	**i**	$\frac{1}{2}Mv^2 = Mgh$ ✓ $h = (L - L\cos\theta)$ ✓ Result follows	**2**
b	**ii**	$Mg\cos\theta = \frac{Mv^2}{R}$ ✓ $Mg\cos\theta = \frac{M \times 2gR(1-\cos\theta)}{R} \Rightarrow \cos\theta = 2 - 2\cos\theta \Rightarrow \cos\theta = \frac{2}{3}$ ✓ $\theta = 48^\circ$ ✓	**3**
c		The angle is independent of the radius ✓ So, no change ✓	**2**

17.

A uniform rod of length L = 8.0 m and weight 15 kN is supported horizontally by a cable attached to a vertical wall. The cable makes an angle of 30° with the horizontal rod. The rod is hinged to the wall.

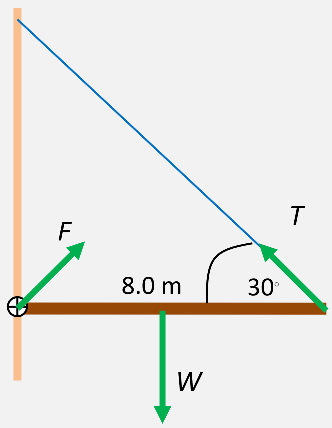
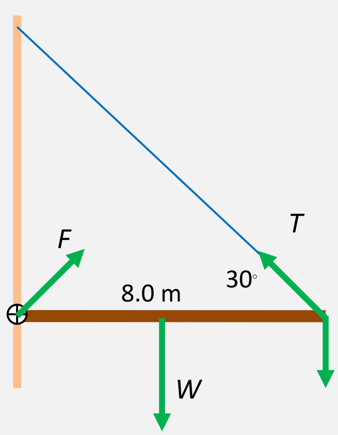


(a) Calculate

(i) the tension in the cable, **[3]**

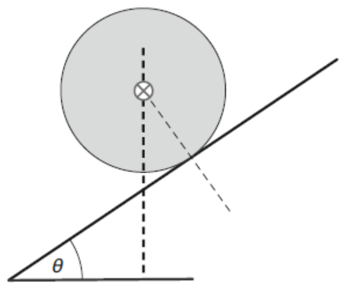
(ii) the magnitude and direction of the force exerted by the wall on the rod. **[3]**

(b) A worker of mass 85 kg can walk anywhere on the rod without fear of the cable breaking. Determine the minimum breaking tension in the cable. **[3]**

Question 17		Answers	Marks
a	i	 The perpendicular distance between the axis at the wall and the line of the tension force is $L\sin 30^\circ$ ✓ Rotational equilibrium by taking torques about an axis through the point of support gives: $W\times\frac{L}{2}=TL\sin 30^\circ$ ✓ $W=T=15\text{ kN}$ ✓	3
a	ii	Translational equilibrium gives: $T\cos 30^\circ=F_x$ and $T\sin 30^\circ+F_y=15\text{kN}$ ✓ $T\cos 30^\circ=F_x=12.99\approx 13\text{kN}$ and $F_y=7.5\text{ kN}$ so $F=\sqrt{12.99^2+7.5^2}=15\text{kN}$ ✓ And the direction to the horizontal is $\theta=\tan^{-1}\frac{7.5}{12.99}=30^\circ$ ✓	3
b		The critical case is when the worker stands all the way to the right ✓  Rotational equilibrium: $W\times\frac{L}{2}+mgL=TL\sin 30^\circ$ ✓ $T_{\text{min}}=16.7\approx 17\text{ kN}$ ✓	3

18.

A cylinder of mass M and radius R rolls down an inclined plane without slipping. The moment of inertia of a cylinder about the axis shown is $\frac{1}{2}MR^2$.

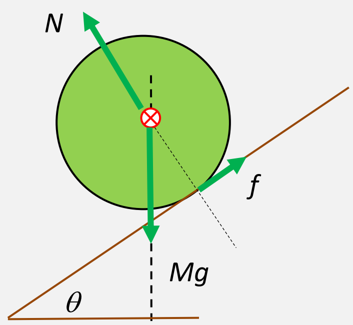


(a) On the diagram above draw arrows to represent the forces acting on the cylinder as it rolls. **[3]**

(b) (i) Show that the linear acceleration of the center of mass of the cylinder is given by $a = \frac{2}{3} g \sin\theta$. **[3]**

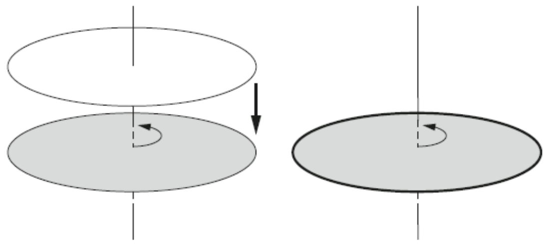
(ii) The mass of the cylinder is M = 12.0 kg and its radius is R = 0.20 m. The incline makes an angle 30° with the horizontal. Determine the frictional force acting on the cylinder. **[1]**

(c) Calculate the rate of change of the angular momentum of the cylinder as it rolls. **[2]**

Question 18		Answers	Marks
a		 weight of the cylinder, Mg ✓ the normal reaction, N ✓ a frictional force f ✓	3
b	i	Newton's second law for the translational motion down the plane is $Mg\sin\theta - f = Ma$ ✓ For the rotational motion by taking torques about the axis through the center of mass is $fR = \frac{1}{2}MR^2\alpha = \frac{1}{2}MR^2 \times \frac{a}{R} = \frac{1}{2}MRa$ ✓ $Mg\sin\theta - \frac{1}{2}Ma = Ma$ ✓ From which the result follows.	3
b	ii	$f = Mg\sin\theta - Ma = 12\times 9.8\times\sin 30^\circ - 12\times\frac{2}{3}\times 9.8\times\sin 30^\circ = 19.6 \approx 20\,\mathrm{N}$ ✓	1
c		The rate of change of the angular momentum is the net torque✓ And this is $fR = 19.6\times 0.20 = 3.92 \approx 4.0\,\mathrm{N\ m}$ ✓	2

19.

A horizontal disc is rotating about a vertical axis through the disc's center of mass. The mass of the disc is 4.00 kg and its radius is 0.300 m. The disc is rotating with an angular velocity of 42.0 rad s^{-1}. A ring of mass 0.500 kg and radius 0.300 m. falls vertically and lands on top of the disc as shown in the diagram. As soon as the ring lands, it slides a bit on the disc and eventually the disc and ring rotate with the same angular velocity.



(a) (i) Explain why the angular momentum of the system before and after the ring lands is the same. **[2]**

(ii) Calculate the final, common angular velocity of the disc-ring system. **[3]**

(iii) Determine the kinetic energy lost as a result of the ring landing on the disc. **[3]**

(b) It took 3.00 s for the ring to start rotating with the same angular velocity as the disc.

(i) Determine the average angular acceleration experienced by the ring during this time. **[1]**

(ii) Calculate the number of revolutions made by the ring during the 3.00 s. **[2]**

(iii) Show that the torque responsible for accelerating the ring to its final constant angular velocity was 0.504 N m. **[2]**

(iv) State and explain, without further calculation, the torque responsible for decelerating the disc. **[2]**

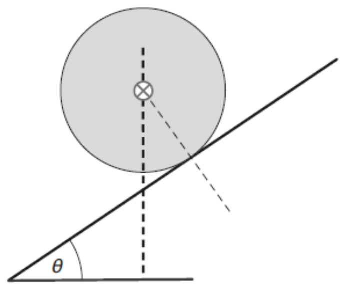
(c) Calculate the average power developed by the torque in (b) (ii) in accelerating the ring. **[2]**

Question 19		Answers	Marks
a	i	When the ring makes contact with the disc and while it is sliding, it exerts a frictional force on the disc but the disc exerts equal and opposite force on the ring✓ Hence the net torque is zero and hence angular momentum is conserved✓	2
a	ii	The initial angular momentum is $L = I\omega = \frac{1}{2}MR^2\omega = \frac{1}{2}\times 4.00\times 0.300^2\times 42.0 = 7.56\text{ J s}$ ✓ After the ring lands the total angular momentum is $L = \frac{1}{2}\times 4.00\times 0.300^2\times\omega' + 0.500\times 0.300^2\times\omega' = 7.56$ ✓ $\omega = 33.6\text{ rad s}^{-1}$ ✓	3
a	iii	The initial kinetic energy is $E_K = \frac{1}{2}I\omega^2 = \frac{1}{2}\times(\frac{1}{2}\times 4.00\times 0.300^2)\times 42.0^2 = 158.76\text{ J}$ ✓ The final is $E_K = \frac{1}{2}\times(\frac{1}{2}\times 4.00\times 0.300^2)\times 33.6^2 + \frac{1}{2}\times(0.500\times 0.300^2)\times 33.6^2 = 127\text{ J}$ ✓ leading to a loss of $158.76 - 127 \approx 31.8\text{ J}$ ✓	3
b	i	$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{33.6}{3.00} = 11.2\text{ rad s}^{-2}$ ✓	1
b	ii	$\Delta\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\times 11.2\times 3.00^2 = 50.4\text{ rad}$ ✓ Which is $\frac{50.4}{2\pi} = 8.0$ revolutions✓	2
b	iii	$\tau = \frac{\Delta L_{\text{ring}}}{\Delta t}$ ✓ $\tau = \frac{0.500\times 0.300^2\times 33.6}{3.00} = 0.504\text{ N m}$ ✓ ***OR*** $\tau = I_{\text{ring}}\alpha$ ✓ $\tau = (0.500\times 0.300^2)\times 11.2 = 0.504\text{ N m}$ ✓	2
b	iv	It is equal and opposite to that on the ring✓ Because the force on the ring is equal and opposite to that on the disc✓	2
c		The change in the kinetic energy of the ring is $\frac{1}{2}\times(0.500\times 0.300^2)\times 33.6^2 = 25.40\text{ J}$ ✓ And so the power developed is $\frac{25.40}{3.00} = 8.47\text{ W}$ ✓ ***OR***	2

		$\bar{P} = \tau \frac{\omega_0 + \omega}{2}$ ✓ $\bar{P} = 0.504 \times \frac{0 + 33.6}{2} = 8.47\ \text{W}$ ✓	

20.

A sphere of mass M and radius R rolls down an inclined plane without slipping. The moment of inertia of a sphere about the axis shown is $\frac{2}{5}MR^2$.



The sphere starts from rest. The center of mass is lowered by a vertical distance H in time T. The incline makes an angle 30° with the horizontal.

(a)

(i) Show, using conservation of energy, that the linear speed of the sphere at time T is $v = \sqrt{\frac{10gH}{7}}$. [3]

(ii) Determine T. [3]

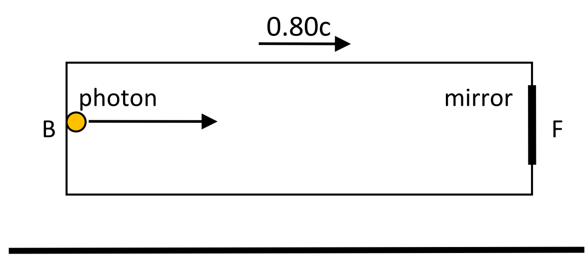
(b) State the angular momentum of the sphere at time T. [1]

(c) Deduce, using the results in (a) and (b), the frictional force on the sphere. [3]

	Question 20	Answers	Marks
a	i	$MgH = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$ ✓ $MgH = \frac{1}{2}\frac{2}{5}MR^2\frac{v^2}{R^2} + \frac{1}{2}Mv^2 = \frac{7}{10}Mv^2$ ✓ $v^2 = \frac{10gH}{7}$ ✓	3
a	ii	Distance travelled down the incline is $2H$ ✓ $s = \frac{u+v}{2}T \Rightarrow 2H = \frac{0+v}{2}T$ ✓ $T = \frac{4H}{v} = \frac{4H}{\sqrt{\frac{10gH}{7}}} = 4\sqrt{\frac{7H}{10g}}$ ✓	3
b		$L = I\omega = \frac{2}{5}MR^2\frac{v}{R} = \frac{2}{5}MR\sqrt{\frac{7gH}{10}}$ ✓	1
c		$\tau = \frac{\Delta L}{\Delta t} = \frac{\frac{2}{5}MR\sqrt{\frac{10gH}{7}}}{4\sqrt{\frac{7H}{10g}}}$ ✓ $\tau = \frac{MgR}{7}$ ✓ $\tau = fR \Rightarrow f = \frac{Mg}{7}$ ✓	3

21.

A rocket moves at a speed of 0.80c relative to the ground. A photon is emitted from the back of the rocket (B) and, after reflection from a mirror at the front of the rocket (F), returns to its emission point 3.0 μs later as measured by the rocket observers.



(a)
(i) Suggest why this time interval measured by the rocket observers is a proper time interval. **[1]**
(ii) Calculate this time interval as measured by the ground observers. **[2]**

(b) Determine
(i) the proper length of the rocket, **[2]**
(ii) the length of the rocket measured by the ground. **[1]**

(c) (i) Show that the time it takes the photon to travel from B to F in the ground frame is 4.5 μs. **[1]**
(ii) Comment on your answer in view of the answer to (a) (ii). **[2]**

(d) (i) Calculate the invariant spacetime interval for the events E_1: the photon leaves B and E_2: the photon arrives at F. **[1]**

(ii) Hence or otherwise calculate the distance travelled by the photon in getting from B to F according to the ground observers. **[2]**

Question 21		Answers	Marks
a	i	The emission and reception of the photon happen at the same point in space for the rocket observers.✓	1
a	ii	$\gamma = \frac{1}{\sqrt{1-0.80^2}} = \frac{5}{3}$ ✓ $\Delta t = \gamma(\Delta t' + \frac{v}{c}\Delta x') = \frac{5}{3} \times (3.0 \times 10^{-6} + 0) = 5.0\ \mu s$ ✓	2
b	i	$L_0 = \frac{3.0 \times 10^{-6}}{2} \times 3.0 \times 10^8 = 450\ m$ ✓	1
b	ii	Length measured by ground $\Delta x = \frac{450}{\frac{5}{3}} = 270\ m$ ✓	1
c	i	$t = \gamma(t' + \frac{v}{c^2}x') = \frac{5}{3} \times (1.5 \times 10^{-6} + \frac{0.80}{3.0 \times 10^8} \times 450)$ ✓ $t = 4.5\ \mu s$	1
c	ii	F is moving away from the photon✓ So, the trip from B to F will take longer than from F to B✓	2
d	i	Use the rocket frame to get $(c\Delta t')^2 - (\Delta x)^2 = (3.0 \times 10^8 \times 1.5 \times 10^{-6})^2 - 450^2 = 0$ ✓	1
d	ii	The spacetime interval is zero in the ground frame as well✓ $0 = (3.0 \times 10^8 \times 4.5 \times 10^{-6})^2 - \Delta x^2 = 0$ so $\Delta x = 1350\ m$ ✓ ***OR*** $x = \gamma(x' + vt') = \frac{5}{3} \times (450 + 0.80 \times 3.0 \times 10^8 \times 1.5 \times 10^{-6})$ ✓ $x = 1350\ m$ ✓	2

22.

(a) State the postulates of special relativity. **[2]**

Two identical spacecraft, A and B, are approaching each other with speed 0.600c as measured by an observer on the ground.

(b) Calculate the velocity of B relative to A according to

(i) Galileo, **[1]**

(ii) Einstein. **[2]**

(c) An observer on spacecraft A measures that the length of B is 1.20×10^2 m. Determine the proper length of the spacecraft. **[2]**

Two events are separated in space by $\Delta x = x_L - x_E = 1.20\times10^3$ m and in time by $\Delta t = t_L - t_E = 3.00\,\mu s$ in the ground frame. (In other words L occurs *after* E in this frame.)

(d)

(i) Calculate the separations in space and time of these events in the frame of rocket A. **[3]**

(ii) Explain why in the ground frame, event E *cannot* be the cause of event L. **[2]**

(iii) Show that there are reference frames in which L occurs *before* E. **[3]**

Question 22		Answers	Marks
a		All laws of Physics are the same in all inertial reference frames✓ The speed of light in vacuum is the same in all inertial reference frames✓	2
b	i	− 1.20c✓	1
b	ii	$u' = \dfrac{u-v}{1-\dfrac{uv}{c^2}} = \dfrac{-0.60c-0.60c}{1-\dfrac{(-0.60c)(0.60c)}{c^2}}$ ✓ $u' = \dfrac{u-v}{1-\dfrac{uv}{c^2}} = \dfrac{-1.20c}{1+0.36} = -0.88235c \approx -0.882c$ ✓	2
c		$\gamma = \dfrac{1}{\sqrt{1-0.88235^2}} = 2.125$ ✓ $L = \dfrac{L_0}{\gamma} \Rightarrow L_0 = \gamma L = 2.125\times120 = 255\text{ m}$ ✓	2
d	i	$\gamma = \dfrac{1}{\sqrt{1-0.60^2}} = 1.25$ ✓ $\Delta x' = \gamma(\Delta x - v\Delta t) = 1.25\times(1200-0.60\times3\times10^8\times3.0\times10^{-6}) = 825\text{ m}$ ✓ $\Delta t' = \gamma(\Delta t - \dfrac{v}{c^2}\Delta x) = 1.25\times(3.00\times10^{-6} - \dfrac{0.60c}{c^2}\times1200) = 0.750\ \mu\text{s}$ ✓	3
d	ii	A possible signal from E to L causing L would have a speed $\dfrac{1200}{3.0\times10^{-6}} = 4.0\times10^8\text{ m s}^{-1}$ ✓ And so is impossible since no signal can travel faster than light in vacuum✓	2
d	iii	We need to have $\Delta t' = \gamma(\Delta t - \dfrac{v}{c^2}\Delta x) < 0$ ✓ We need to have $3.0\times10^{-6} - \dfrac{v}{c^2}\times1200 < 0 \Rightarrow v > \dfrac{3.0\times10^{-6}}{1200}c^2$ ✓ $v > 2.25\times10^8\text{ m s}^{-1}$ ✓ So in frames with speed greater than the above L occurs before E.	3

23.

A spacecraft leaves earth with a speed of 0.60c towards a space station 6.0 ly away (according to earth).

(a) Calculate the time the spacecraft will reach the space station according to

(i) earth observers, **[1]**

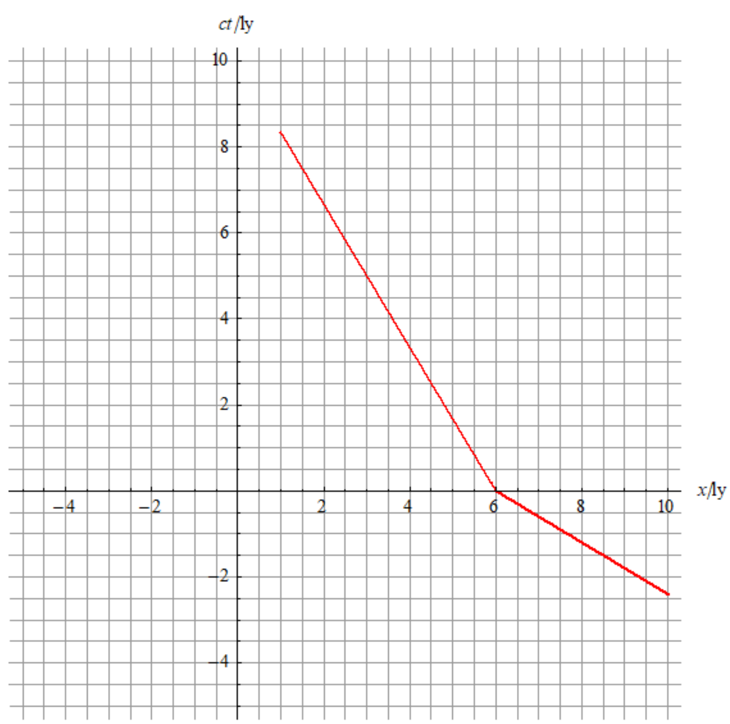
(ii) spacecraft observers. **[2]**

(b) When the spacecraft left earth, a probe left the space station with speed 0.60c on its way back to earth. Probe, spacecraft and earth clocks show zero at departure. The probe sends a radio signal to earth announcing their departure.
Calculate the time it will take the signal to arrive on earth according to

(i) earth observers, **[2]**

(ii) probe observers. **[4]**

(c) The spacetime diagram shows the axes of the frame of the earth and, in red, the axes of the probe.



On the axes,

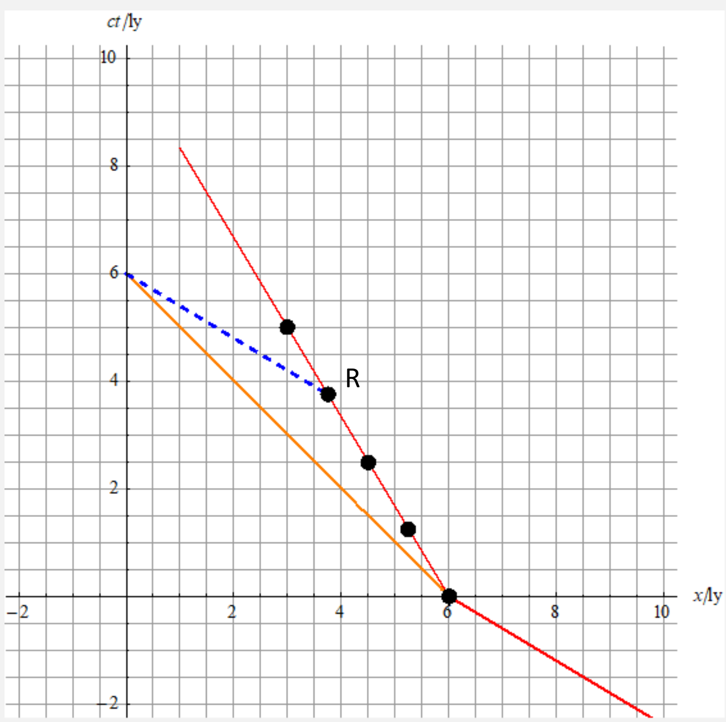
(i) draw the worldline of the emitted photon by the probe, **[1]**

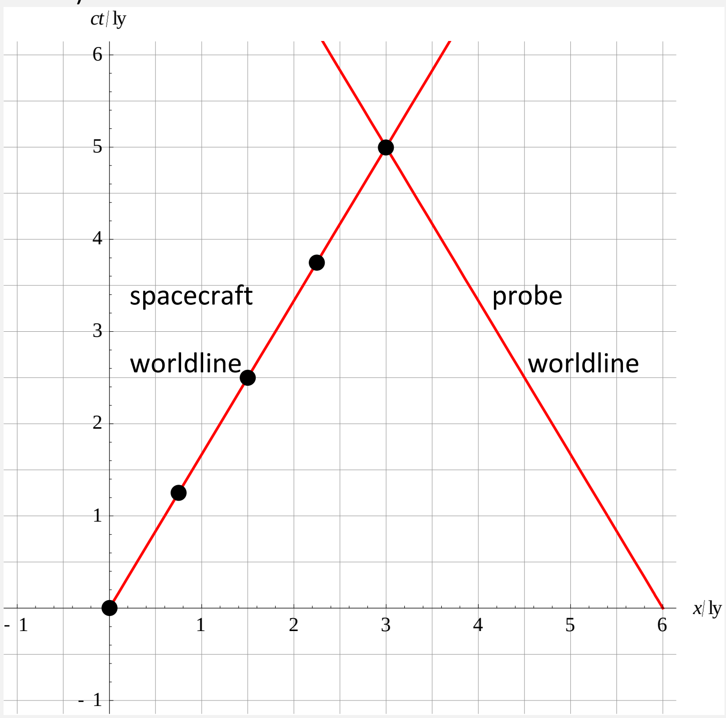
(ii) by drawing a suitable line, label with the letter R the point on the primed time axis that gives the time of the event “signal is received at earth”. **[1]**

(d) The probe and spacecraft will meet along the way. Determine the time this happens according to

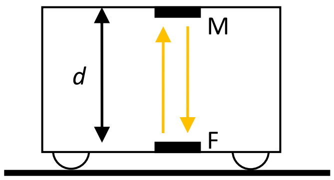
(i) earth observers, **[1]**

(ii) spacecraft observers. **[2]**

Question 23		Answers	Marks
a	i	$\frac{6.0\,\text{ly}}{0.60c} = 10\text{ yr}$ ✓	1
a	ii	The gamma factor is 1.25✓ Hence the time is $\frac{10}{1.25} = 8.0\text{ yr}$ ✓	2
b	i	The signal moves at the speed of light✓ $\frac{6.0\,\text{ly}}{c} = 6.0\text{ yr}$ ✓	2
b	ii	According to the probe the distance separating the earth and the probe at the time of emission of the signal is $\frac{6.0}{1.25} = 4.8\,\text{ly}$ ✓ In the time *T* it takes the signal to get to earth, the earth moves closer to the probe by a distance $0.60cT$✓ Hence $cT = 4.8 - 0.60cT$ ✓ $T = \frac{4.8\,\text{ly}}{1.60c} = 3.0\text{ yr}$ ✓	4
c	i	The dots on the primed time axis are separated by 1 year by probe clocks.  Orange line ending at $x = 0$, $ct = 6$✓	1
c	ii	Blue dashed line starting from $x = 0$, $ct = 6$ parallel to primed space axis arriving at R at 3 yr on probe clocks✓	1
d	i	$\frac{6.0\,\text{ly}}{1.20c} = 5.0\text{ yr}$ ✓	1

d	**ii**	For earth, the spacecraft meets the probe at $x = 3.0$ ly at $t = 5.0$ yr ✓ Hence $t' = \gamma(t - \frac{v}{c^2}x) = \frac{5}{4} \times (5.0 - \frac{0.60c}{c^2} \times 3.0) = 4.0$ yr ✓ ***OR*** Using a spacetime diagram: (dots are one year apart according to spacecraft clocks)  	**2**

24.
An observer inside a train carriage at rest relative to the ground performs an experiment in which a beam of light emitted from the floor, F, of the carriage, gets reflected from a mirror, M, at the ceiling and returns to the point F of emission at the floor. The light returns to F after time τ .



(a) (i) Write down an expression for the speed of light measured by the train observer in terms of *d* and τ . **[1]**

(ii) The carriage now moves with speed *v* towards the right relative to the ground. Suggest whether the answer to (i) changes. **[1]**

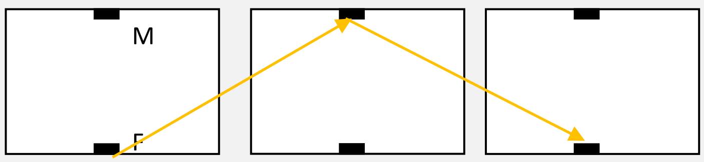
(b) Draw, in the space below, the path of the light according to the observer on the ground, from the time the light is emitted until it returns to F. **[1]**

(c) The following data are available: $v = 0.80c$ and $\tau = 24\text{ ns}$.

Calculate the time it takes light to go from F to M and back to F, the ground observer would measure according to
(i) Galilean relativity, **[1]**
(ii) Einstein relativity. **[2]**

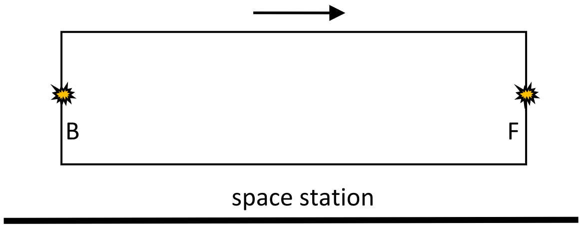
(d) The pion is an unstable particle. The lifetime of a pion at rest is 26 ns. A pion is created in a lab at point C and moves with speed 0.95c relative to the lab. A detector is placed at point D where the pion is expected to decay.

(i) Calculate the distance CD. [3]
(ii) An observer is travelling with the pion. Explain how this observer also measures that the pion has decayed at D. [3]

Question 24		Answers	Marks
a	**i**	$c=\dfrac{2d}{\tau}$ ✓	**1**
a	**ii**	No change ✓	**1**
b		 	**1**
c	**i**	24 ns✓	**1**
c	**ii**	$\gamma=\dfrac{1}{\sqrt{1-0.80^2}}=\dfrac{5}{3}$ ✓ $t=\gamma\tau=\dfrac{5}{3}\times 24=40\text{ ns}$ ✓	**2**
d	**i**	The gamma factor is $\gamma=\dfrac{1}{\sqrt{1-0.95^2}}=3.203$ ✓ Lifetime in lab is $3.203\times 26\times 10^{-9}=8.33\times 10^{-8}\text{ s}$ ✓ Distance is $vt=0.95\times 3.0\times 10^8\times 8.33\times 10^{-8}=23.73\approx 24\text{ m}$ ✓	**3**
d	**ii**	The distance between C and D is contracted to $\dfrac{23.73}{3.203}=7.409\text{ m}$ ✓ The detector is moving towards this observer and will meet in $\dfrac{7.409}{0.95\times 3.0\times 10^8}=26\text{ ns}$ ✓ Hence the pion will decay at D✓	**3**

25.

(a) A rocket of proper length 930 m moves past a space station with speed 0.60c.

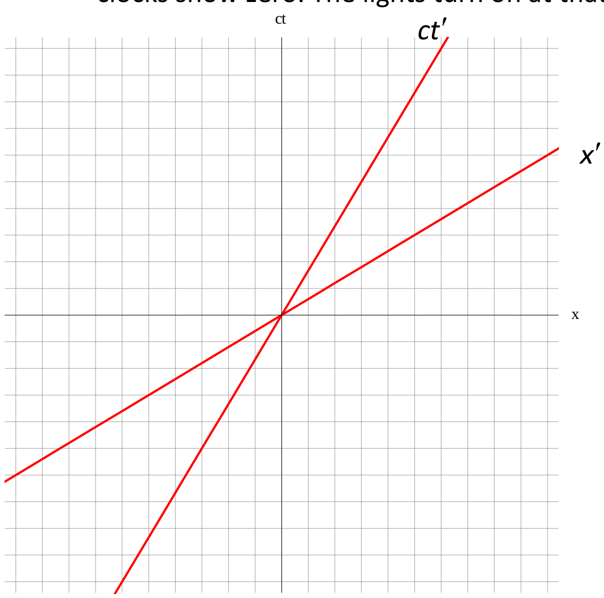


Two lights, at the back (B) and front (F) of the rocket, turn on at the same time according to rocket clocks.
For space station observers

(i) Determine, without any calculations, which light turns on first. **[3]**

(ii) Calculate the time difference between the lights turning on. **[2]**

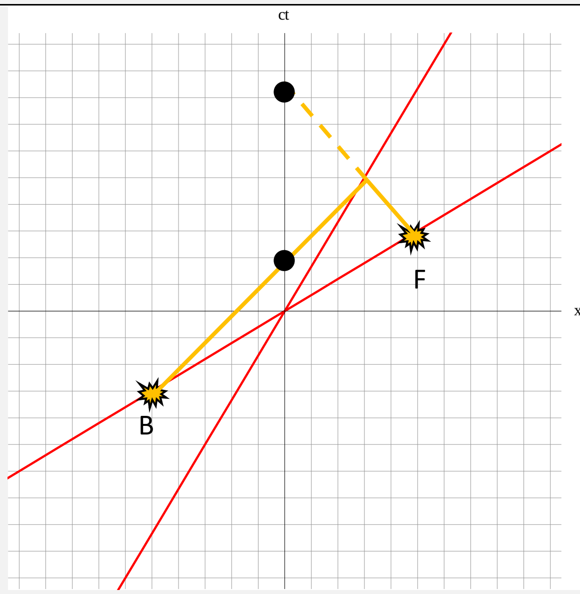
(b) The spacetime diagram shows the frame of the space station and that of the rocket. The instant the middle of the rocket is over the middle of the space station the origins coincide and all clocks show zero. The lights turn on at that instant according to the **rocket**.



(i) On the diagram, label the events representing the lights turning on, using the letters B and F (no numbers are required), **[1]**

(ii) By drawing appropriate lines on the diagram show that light from B and F reaches the observer in the middle of the rocket at the same time, **[1]**

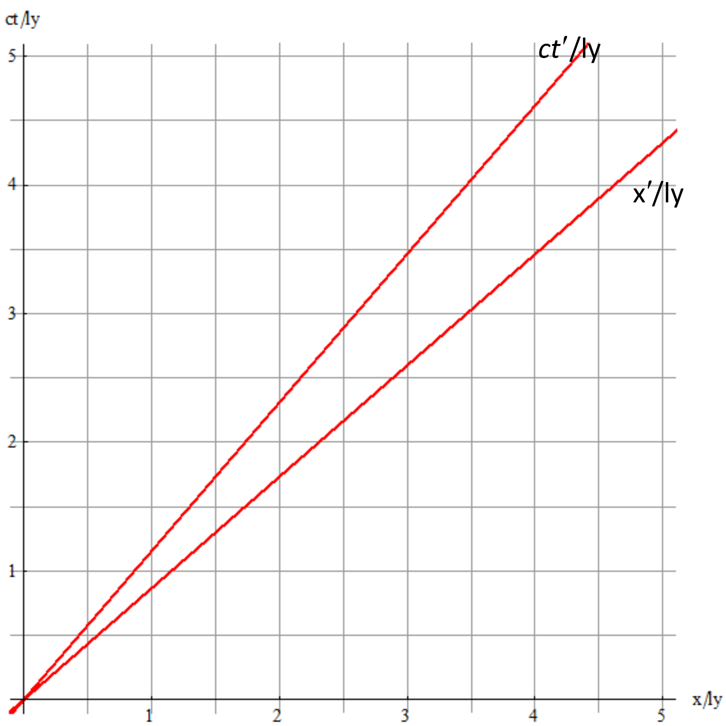
(iii) Using the lines you drew in (ii) show that light from B and F arrive at different times at the observer in the middle of the space station. **[1]**

Question 25		Answers	Marks
a		An observer R in the middle of the rocket will receive light from B and F at the same time✓ The arrival of light from B and F is also simultaneous for a space station observer S✓ But S also sees that R is moving away from the light from B. Since the speed of light is constant the light from B was emitted first ✓	3
b		$\Delta t = t_F - t_B = \gamma(\Delta t' + \frac{v}{c^2}\Delta x') = 1.25 \times (0 + \frac{0.60c}{c^2} \times 930)$ ✓ $\Delta t = t_F - t_B = +2.3\,\mu\text{s}$ ✓	2
c		 Points B and F equidistant from O on the primed space axis✓ Lines at 45° starting from B and F in opposite directions✓ Extension of lines in (ii) to intersect time axis✓	3

26.

(a) Two events are simultaneous in an inertial frame S and happen at the same point in space. Show that these events are also simultaneous in any other inertial frame S' moving past S. **[1]**

(b) The diagram shows the axes for frame S and those for frame S'. Frame S' is a spacecraft moving away from Earth.



(i) Determine the speed of the spacecraft relative to S. **[1]**

(ii) Mark on the diagram the points with coordinates ($x' = 1$ ly , $ct' = 0$) and($x' = 0$, $ct' = 1$ ly). **[3]**

(c) When one year goes by according to spacecraft clocks a light signal is sent to Earth.

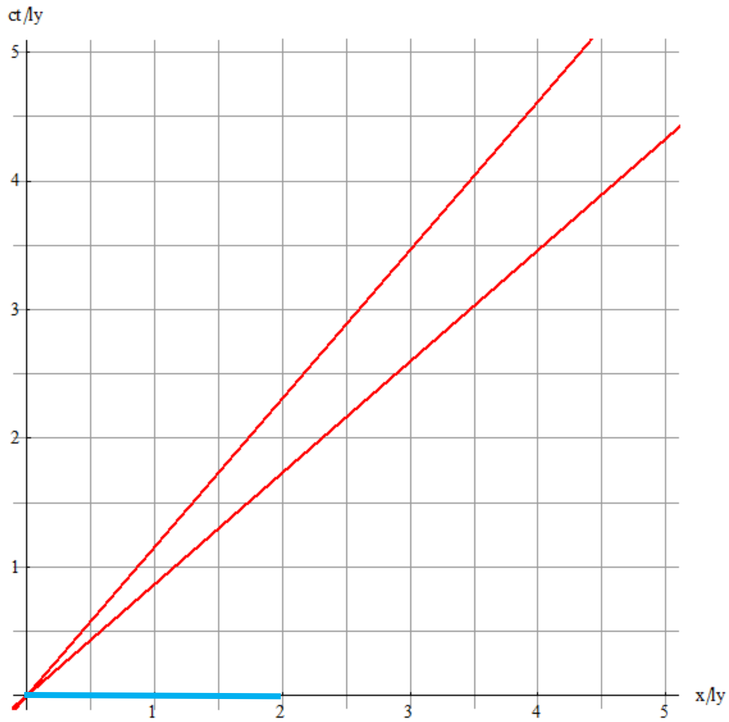
Use the diagram to estimate the time at which the signal is received on Earth, according to Earth. **[2]**

(d) A space station, at rest relative to the Earth at x = 5.0 ly sends a light signal to the spacecraft when ct = 2.5 ly. Use the diagram to determine the time the spacecraft receives the signal according to

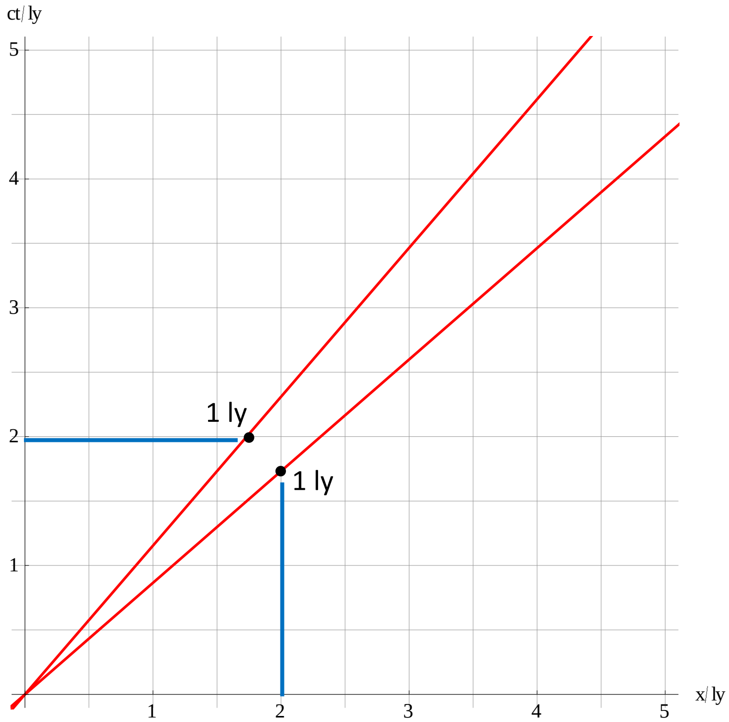
(i) spacecraft clocks **[1]**

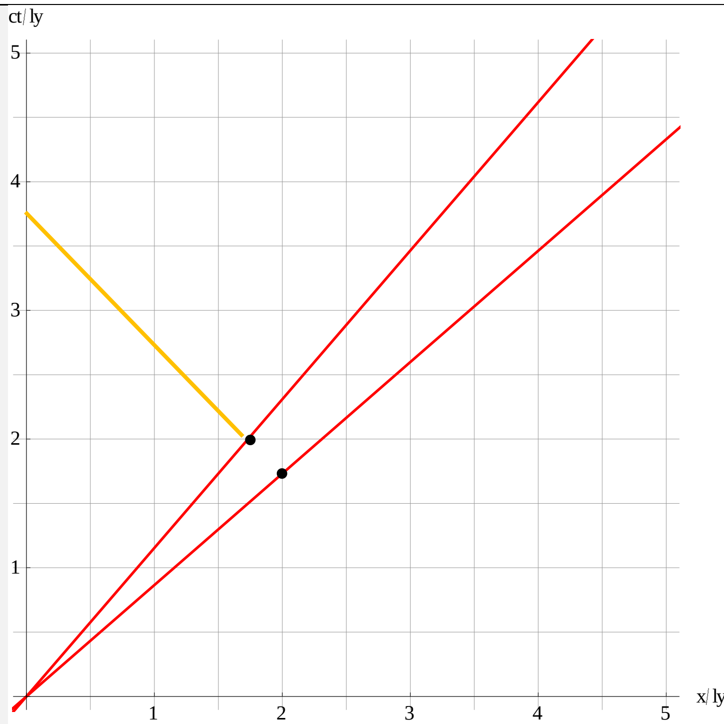
(ii) Earth clocks **[1]**

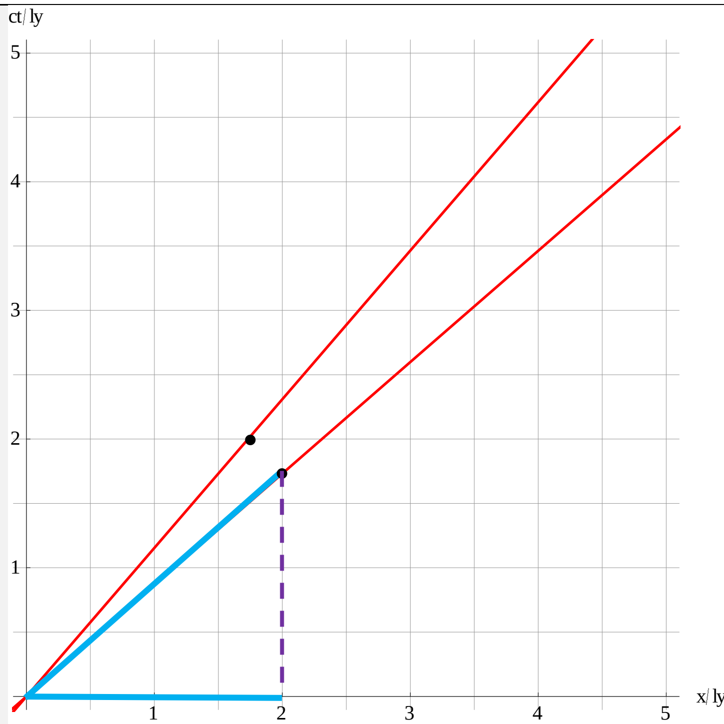
(e) The blue line represents a distance of 2.0 ly measured in S.



Draw a line to represent this distance as measured in S'. **[2]**

Question 26		Answers	Marks
a		$\Delta t' = \gamma(\Delta t - \frac{v}{c^2}\Delta x) = \gamma(0 - \frac{v}{c^2} \times 0) = 0$ ✓	1
b	i	$\frac{v}{c} \approx \frac{3}{3.5} = 0.86$ ✓	1
b	ii	$\gamma = \frac{1}{\sqrt{1-0.86^2}} \approx 2$ ✓ $(x' = 1\,\text{ly},\ ct' = 0)$: $x = \gamma(x + vt) = \gamma(1 + v \times 0) = \gamma = 2.0$ ✓ $(x' = 0,\ ct' = 1\,\text{ly})$: $ct = \gamma(ct' + \frac{v}{c}x') = \gamma(1 + \frac{v}{c} \times 0) = \gamma = 2.0$ ✓  	3
c			2

		 Line as shown✓ *ct* = 3.7 ly approximately so 3.7 years ✓	
d		 Spacecraft: 2.0 y✓ Earth: 4.0 y✓	**2**
e			**2**



Vertical line from $x = 2$ ly✓

Intersecting primed space axis at $x' = 1.0$ ly ✓